# 5-1: Probability Distributions

## Objective 1. Construct a Probability Distribution for a Random Variable.

A \_\_\_\_\_\_\_\_\_\_ is a characteristic or attribute that can assume different values.

A \_\_\_\_\_\_\_\_\_ variable is a variable whose values are determined by chance.

\_\_\_\_\_\_\_\_\_\_ variables are variables that have a finite number of possible values or an infinite number of values that can be counted.

Variables that can assume all values in the interval between any two given values are called \_\_\_\_\_\_\_\_\_\_ variables. Continuous random variables are obtained from data that can be \_\_\_\_\_\_\_\_\_\_\_\_\_ rather than counted.

### Probability Distributions

A probability \_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a process that leads to well-defined results or outcomes due to chance. A \_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ consists of the values of a random variable can assume and the corresponding probabilities of the values the probabilities are determined theoretically or by observation.

### Examples Based on Theory

### **Example 5-1. Probability Distribution for Tossing Three Coins.**

Construct a probability distribution for a discrete random variable using a probability experiment of tossing three coins.

Toss three coins. The sample space is {TTT, TTH, THT, HTT, HHT, HTH, THH, HHH}. If *X* be the random variable of the number of heads, then *X* assumes values of 0, 1, 2, or 3.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Heads | No Heads | One Head | Two Heads | Three Heads |
|  | TTT | TTH, THT, HTT | HHT, HTH, THH | HHH |
| *X* | 0 | 1 | 2 | 3 |
| Probability | \_\_/8 | \_\_/8 \_\_/8 \_\_/8 | \_\_/8 \_\_/8 \_\_/8 | \_\_/8 |
| P(*X*) | \_\_/8 | \_\_/8 | \_\_/8 | \_\_/8 |

|  |  |
| --- | --- |
| ***X*** | ***P*(*X*)** |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

### **Example 5-2. Probability Distribution for Rolling a Single Die**

Construct the probability distribution for rolling a single fair die.

The sample space for a single die is 1, 2, 3, 4, 5, 6, with each outcome having equal probability of 1/6.

|  |  |
| --- | --- |
| ***X*** | ***P(X)*** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

### Example Based on Observation

### **Example 5-3. Number of World Series Games**

The baseball World Series is played by the winner of the National League and the American League. The first team to win four games wins the World Series. In other words, the series will consist of four to seven games, depending on the individual victories. The data shown consist of 40 World Series events. The number of games played in each series is represented by the variable *X*. Find the probability P(*X*) for each *X* and construct a probability distribution. For the 40 World Series included, the winner was determined in four games 8 time, five games 7 times, six games 9 times, and seven games 16 times.

*Solution:*

|  |  |
| --- | --- |
| *X* | P(*X*) |
| 4 | \_\_/40 = \_\_\_\_ |
| 5 | \_\_/40 = \_\_\_\_ |
| 6 | \_\_/40 = \_\_\_\_ |
| 7 | \_\_/40 = \_\_\_\_ |

### Requirements for a Probability Distribution

1. The sum of the probabilities of all the events in the sample space must equal \_\_\_\_. (Formula: \_\_\_\_\_\_\_\_\_)
2. The probability of each event in the sample space, P(X), must be between or equal to \_\_ and \_\_\_. (Formula: \_\_\_\_\_\_\_\_\_\_\_\_).

The sum of the probabilities of all events must be equal to \_\_\_\_\_ since the sample space includes all possible outcomes of the probability experiment.

The probability of any individual event must be a value from \_\_\_ to \_\_\_. A probability cannot be a \_\_\_\_\_\_\_\_\_\_ number or greater than \_\_\_.

# 5 – 2 Mean, Variance, Standard Deviation, and Expectation

## Objective 2. Find the Mean, Variance, Standard Deviation, and Expected Value for a Discrete Random Value.

The mean, variance, and standard deviation for a probability distribution are comprised differently from the mean, variance, and standard deviation for samples.

### Mean

How would you compute the mean number of spots showing on top of a die when it is rolled? How many times would it have to be rolled? Try the mean of 10 rolls, 100 rolls, and 200 rolls. Each is only a sample approximating the mean. You would have to roll the die an \_\_\_\_\_\_\_\_\_\_\_ number of times! Now, that’s impossible! So the formulas used before cannot be used.

### **Example 5-4. Tossing Coins**

Suppose we consider tossing a coin two times. List the sample space?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

In the long run you would expect two heads to occur about \_\_\_\_\_ of the time; one head and one tail about \_\_\_\_\_\_\_ of the time; and no heads (that is, two tails) about \_\_\_\_\_\_\_ of the time. So, on average, the number of heads (mean of the distribution) would be \_\_\_\_\_\_.

Look at the probability distribution of the number of heads when a coin is tossed an infinite number of times.

| Number of Heads *(X)* | 0 | 1 | 2 |
| --- | --- | --- | --- |
| Probability *P(X)* |  |  |  |

Now, find the mean of the probability distribution to confirm the expected or long time average number of heads.

*Solution:*

μ = Σ (*X* ⋅ *P*(*X*)) = 0 ⋅ *P*(0) + 1 ⋅ *P*(1) + 2 ⋅ *P*(2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(Rounding rule for mean, variance, and standard deviation of a Probability Distribution: Round the mean, variance, and standard deviation to one more decimal place than the outcome *X*.

### Formula for the Mean of a Probability Distribution

The mean of a random variable with a discrete probability distribution is

,

where X1,X2, X3, … , Xn are the outcomes and P(X1), P(X2), P(X3), … P(Xn)are the corresponding probabilities.

**Note:**  means to sum the products.

### **Example 5-5. Rolling a Die**

Use a probability distribution to find the mean number of spots that appear when a fair die is tossed. The mean is \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

| Outcome *X* | 1 | 2 | 3 | 4 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- |
| Probability *P(X*) |  |  |  |  |  |  |

or \_\_\_\_\_.

Why is the probability for each of the probabilities be ?

### **Example 5-6. Number of Trips of Five Nights or More**

A study concerning the number of trips of five nights or more that American adults take per year showed that 6% did not take any trips lasting five nights or more, 70% take one trip lasting five nights or more, 20% take two trips lasting five nights or more, 3% take three trips lasting five nights or more, while only 1% take four trips lasting five nights or more. Fill in the chart below and find the mean number of trips American adults take per year.

| Number of trips, *X* | 0 | 1 | 2 | 3 | 4 |
| --- | --- | --- | --- | --- | --- |
| Probability, *P*(*X*) |  |  |  |  |  |

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

Thus, the mean number of trips lasting more than five nights or more per year taken by American adults is \_\_\_\_\_\_\_\_\_\_.

### Rounding Rule

The mean, variance, and standard deviation should be rounded to one more decimal place than the outcome, *X*. When fractions are used, they should be simplified to lowest terms.

### Variance and Standard Deviation

The mean gives a long-run average, but does not describe the spread of the distribution. Thus, we need to calculate the variance and standard deviation. he formulas from Chapter 3 cannot be used for a random variable of a probability distribution, so we use a different formula:

, and the standard deviation is

.

This formula is tedious, so we use a simpler, algebraically equivalent formula, or technology.

### Formula for Variance (and Standard Deviation) of a Probability Distribution

The simpler formula for the variance is .

Thus the standard deviation is .

Notice that variance and standard deviation are always positive.

### Example 5-7. Rolling a Die

Use a probability distribution to find the mean number of spots that appear when a fair die is tossed. Earlier we found that the mean of this probability distribution was 3.5. Find the variance and standard deviation.

| Outcome *X* | 1 | 2 | 3 | 4 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- |
| Probability *P*(*X*) |  |  |  |  |  |  |

.

.

### Example 5-8. Number of Trips Lasting Five Nights or More

Using the chart from in the example in which we found the mean number of trips lasting five nights or more that American adults take per year, find the variance and standard deviation of the distribution.

| Number of trips, *X* | 0 | 1 | 2 | 3 | 4 |
| --- | --- | --- | --- | --- | --- |
| Probability *P*(*X*) | 0.06 | 0.70 | 0.20 | 0.03 | 0.01 |

Recall that the mean number of trips lasting more than five nights or more per year taken by American adults is \_\_\_\_\_\_\_\_\_\_.

Use the distribution in vertical columns finding *X*× P(*X*) and *X2*× P(*X*) for each outcome, *X*, and probability, P(*X*), then summing the columns and using the results in the formula.

| *X* | *P*(*X*) | *X*× *P*(*X*) | *X*2× *P*(*X*) |
| --- | --- | --- | --- |
| 0 | 0.06 |  |  |
| 1 | 0.70 |  |  |
| 2 | 0.20 |  |  |
| 3 | 0.03 |  |  |
| 4 | 0.01 |  |  |
| Totals | |  |  |

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_

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### Using Your TI-84

First, press <STAT> and select 1: Edit…

Enter the values of the outcomes, *X*, into List 1 (L1) and the values of the probabilities, *P*(X), into List 2 (L2).

Values of the variable are entered in List 1.  Then moving to List 2 to enter the probabilities. Probabilities for each value of the variable are entered into List 2.

Then, press **<STAT>,** use the right arrow to choose **CALC**, and select **1:1-Var Stats**.

Enter L1 for the List and L2 for the FreqList and calculate.

Screen shot showing the menu for the frequency chart entered into List 1 and LIst 2.  STAT, CALC, 1:i-Var-Stat with the List as L!, the FreqList as L2, and Calculate. Screen shot shows the report for find the mean, standard deviation and other statistics as calculated for the frequency chart using the TI-83/84.

The mean is 1.23, or 1.2 when rounded, and the standard deviation is .6458.

To find the variance, square the standard deviation.

### Expectation

Expected value, or expectation, is used in games of chance, insurance, and other areas such as decision theory. **Expected value**, denoted by ***E*(*X*),** is another name for the mean of the distribution. Thus the formula is .

In gambling games, an expected value of \_\_\_\_\_ indicates a fair game. If the expected value is \_\_\_\_\_\_\_\_\_\_\_\_\_\_, the game favors the house and the player will, in the long run, lose money. If the expected value is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, the game favors the player and the player will, in the long run, win money.

### Example 5-9. A Dice Game

A person pays $3 to play a certain game by rolling a single die once. If a 1, 2, or 3 comes up, the person wins nothing. If, however, the player rolls a 4, 5, or 6, he or she wins the difference between the number rolled and $3. Find the expectation for this game. Is the game fair?

Find the gain and probability for each roll:

| Roll | 1 | 2 | 3 | 4 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- |
| Gain *X* | 3 | 3 |  | 1 |  |  |
| Probability *P*(*X*) |  |  |  |  |  |  |

*E*(*X*) = + = \_\_\_\_\_\_\_

Thus, in the long run, a player will \_\_\_\_\_\_\_ money and the game (is) / (is not) fair.

### Example 5-10. Insurance

An insurance company insures a person’s antique coin collection worth $20,000 for an annual premium of $300. If the company figures that the probability of the collection being stolen is 0.002, find the company’s expected profit.

(Construct a table showing gains or losses and the probability of each to determine the expected value.)

The expected value is $\_\_\_\_\_\_\_.

# 5 – 3 The Binomial Distribution

## Objective 3. Find the Exact Probability for *X* Successes in *n* Trials of a Binomial Experiment.

Some variables can have only two outcomes, or the outcomes can be classified in exactly two categories. For example, when a coin is tossed, it will land on heads or tails. In addition, a multiple-choice question with five options will have one correct choice and the other four choices will be incorrect. Thus, the answer choices can be classified as correct or incorrect. These situations are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Each repetition of the experiment is called a trial.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a probability experiment that satisfies the following four requirements:

1. There must be a \_\_\_\_\_\_\_\_\_\_ number of trials.
2. Each trial can have only \_\_\_\_\_\_\_ outcomes or outcomes that can be reduced to \_\_\_\_\_\_ categories. These outcomes can be considered either success or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
3. The outcomes of each trial must be \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of one another.
4. The probability of a success must remain the \_\_\_\_\_\_\_\_\_ for each trial.

A success in a binomial experiment is neither good nor bad. A success just means that the result was what the researcher was looking for. For example, a multiple choice test is given and the outcomes are either correct or incorrect. If the researcher is looking for the probability of incorrect answers, then an incorrect answer would be considered a *success*. On the other hand, if the researcher were looking for the probability of correct answers, then a correct answer would be considered a *success*.

### Example 5.11. Classify as Binomial or Not

Classify each of the situations as binomial or not.

1. Roll a standard die 200 times.
2. Survey 100 shoppers asking for their favorite exercise shoe brand.
3. Survey 100 teenagers asking if they play the video game *Halo*.
4. Ask 300 viewers if they recall seeing a Coca Cola commercial during a particular show?
5. Test four brands of aspirin to see which brands are effective.

*Solution:*

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

### Notation Used for Binomial Experiments

*n* = the number of trials

*X =* the number of successes in *n* trials (*X* is a counting number: 0, 1, 2, 3, … , *n*)

*P(S) =* the probability of success

*P(F) =* the probability of failure

*p =* the numerical probability of success; *P(S) = p*

*q =* the numerical probability of failure; *P(F)* = 1 – *p = q*

### Binomial Probability Formula

In a binomial experiment, the probability of exactly *X* successes in *n* trials is

*nCX*

*nCX =*

Keep in mind that no successes (*X* = 0) must be considered.

### Example 5-12. Probability of Rolling 2’s

A 6-sided die is tossed 5 times. A success is tossing a 2. A failure is tossing a result that is not a 2. Each toss of the die is independent of any other toss.

and

1. Find the probability of getting exactly 3 2’s.
2. Find the probability of at least 3 2’s.
3. Find the probability of at most 2 2’s.

*Solution:*

Consider the probabilities for each possible number of outcomes:

| *X* | P(*X*) |
| --- | --- |
| 0 | 4019 |
| 1 | 4019 |
| 2 | 1608 |
| 3 | 0322 |
| 4 | 0032 |
| 5 | 0001 |

0322

0355

9645

### Example 5-13. Probability of Too Much Concern

In a survey, 3 of 4 students said the courts show “too much concern” for criminals. Find the probability that at most 3 out of 7 randomly selected students will agree with this statement.

*Solution:*

### Using Binomial Probability Tables (in text, Appendix A, Table B)

Table B from the text is the Binomial Distribuiton Table for n = 2, 3, 4, 5, 6, and 7 which gives the probability, rounded to four decimal places, for X = 0 to n for probabilities 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.95.

### Example 5-14. Probability of Wanting a Toy

If 20% of children were reported to want a particular toy, use the table to find the probability that

1. exactly 4 of 7 children want that toy.
2. at most 3 of 7 children want that toy.
3. at least 4 of 7 children want that toy.

*Solution:*

## Objective 4. Find the Mean, Variance, and Standard Deviation for the Variable of a Binomial Distribution.

The mean, variance and standard deviation for a binomial variable can be found using the methods for any probability distribution, but there are shorter, mathematically equivalent formulas for the mean, variance and standard deviation for binomial distributions.

Mean:

|  | The mean is equal to the product of the number of  trials and the probability of success. |
| --- | --- |

Variance:

|  | The variance is equal to the product of the number of trials, the probability of success and the complement of the probability of success. |
| --- | --- |

Standard deviation:

|  | The standard deviation is equal to the square root of the  variance. |
| --- | --- |

These formulas are especially helpful when, for a binomial experiment, the number of trials is very large.

### Example 5-15. Mean, Variance, and Standard Deviation of Binomial Distribution

For a binomial distribution for which there are 500 trials, and the probability of success is 0.25, what is the mean, variance and standard deviation?

*Solution:*

Mean:

Variance:

Standard deviation:

=

### Example 5-16. Mean and Standard Deviation of Number Purchasing Internet Service

Thirty-two percent of adult Internet users have purchased products or services online. For a random sample of 200 adult Internet users, find the mean and standard deviation for the number who have purchased goods or services online.

*Solution:*

Is this a binomial experiment with 200 trials and a probability of success of 0.32?

There are a fixed number of trials: \_\_\_\_\_\_\_

Each trial has two possible outcomes: \_\_\_\_\_\_\_\_\_\_\_\_\_ or \_\_\_\_\_\_\_\_\_\_\_\_\_

Each trial is independent: \_\_\_\_\_\_\_\_\_\_\_\_

The probability of success remains the same for each trial. \_\_\_\_\_\_\_\_\_\_\_\_

The mean number of adult Internet users, out of 200, who have purchased products or services online is .

The standard deviation of adult Internet users, out of 200, who have purchased projects or services online is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

### Example 5-17. Mean and Standard Deviation of Number of Students

In a survey, 3 of 4 students said the courts show r of 200, who have purchased projects or services online is \_\_\_deviation for a group of 7 randomly selected students who will agree with this statement.

*Solution:*

### Example 5-18. Mean and Standard Deviation of Number of Children

In a survey, 20% of children were reported to want a particular toy. Of 7 children, what is the mean and standard deviation of the number of children who want the toy?

*Solution:*

# 5 – 4 Other Types of Distributions

## Objective 5. Find Probabilities for Outcomes of Variables Using Poisson, Hypergeometric, Geometric, and Multinomial Distributions.

Not all distributions follow the binomial distribution. There are many others.

### Multinomial Distribution

A **multinomial distribution** is a probability experiment, similar to a binomial experiment, for which each trial has a specific, although not always the same, number of outcomes.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a probability experiment that satisfies the following requirements:

1. There must be a \_\_\_\_\_\_\_\_\_\_\_ number of trials.

2. Each trial has a specific number of \_\_\_\_\_\_\_\_\_\_\_\_\_, where the number of outcomes can vary from experiment to experiment, but not trial to trial.

3. The trials are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

4. The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of a particular outcome remains the same.

### Formula for Calculating Probabilities Using the Multinomial Distribution

Shows the formula for hte probability for a multinomial distribution consisting of several events, each having a specific probability of occurring.  The sum of the times each event occurs is the number of trials.  The sum of the probabiltiies of the events is 1. 

### Example 5-19. Multinomial Probability

Suppose we know the probabilities that people choose to go to a movie, to dinner and a play, or shopping are 0.50, 0.30, and 0.20, respectively. Find the probability that of 5 randomly selected people, 3 choose to go to a movie, one chooses to go to dinner and a play, and one chooses to go shopping.

*Solution:*

*n* = 5, *n!* = 120

*X*1 = 3, *X*2 = 1, and *X*3= 1, so *X*1! = 6, *X*2! = 1, and *X*3!= 1

*p1 =* 0.50*, p2 =* 0.30*, p3 =* 0.20

### Poisson Distribution

A Poisson distribution is a discrete probability distribution is useful when the number of occurrences is large, the probability of an even is small, and the independent variable occurs over a long period of time or within a given area or volume.

A Poisson experiment is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ that satisfies the following requirements:

1. The random \_\_\_\_\_\_\_\_\_\_\_\_\_\_, *X*, is the number of occurrences of an event over an interval (of length, area, volume, period of time, etc.)
2. The occurrences occur \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
3. The occurrences are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of one another.
4. The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number of occurrences over an interval is known.

### Formula for Calculating Probabilities Using the Poisson Distribution

The probability of X occurrences in an interval of time, volume, area, and so forth, for a variable where the Greek letter lambda represents the mean number of occurrences per unit of time, volume, area, and so forth, is given where X is equal to 0, 1, 2, and so forth.

We can use Table C in Appendix A in the textbook to find various values of *X,* the number of occurrencesand *λ,* the average number of occurrences per unit*.*

### Example 5-20. Poisson Distribution

In a 400-page manuscript, there are 200 randomly distributed misprints. If a page is selected, find the probability that it has exactly one misprint.

Solution:

There are 400 pages.

X = 1 misprint on the page.

λ = 200/400 = 0.5, the mean number of misprints per page.

P(1:0.5) =

Using the table,

A portion of Table C is included for X equal to 0 to 7 with lambda (the mean number of occurrences) equal to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1.1, 1.2, 1.3, 1.4, 1.5, and 1.6.

we can locate the column where *λ* is 0.5 and the row where *X* is 1.

Notice that the probability shown on the table is also 0.3033

### Example 5-21. Returned Mailings

A mail order company finds that 1.7% of their mailings are returned because of incorrect or incomplete addresses. In a mailing of 200 pieces, find the probability that none are returned. Find the probability that 3 are returned.

*Solution:*

### Hypergeometric Distribution

The **hypergeometric distribution** allows for probabilities when sampling is done without replacement, so the trials are not independent and the binomial distribution does not apply, to be calculated.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a probability experiment that satisfies the following requirements:

1. There are a fixed number of trials.
2. There are two outcomes, and they can be classified as success or failure.
3. The sample is selected without replacement.

### Formula for Calculating Probabilities Using the Hypergeometric Distribution

The formula for the probability of selecting without replacement a sample of size n with X items of type a and n-X items of type b from a population with exactly two types of objects (such as males and females or successes and failures) so that there are a items of one kind and b items of another and a + b equals the total population.

### Example 5-22. Probabilities of Types of Hors D’oeuvres

A plate of hors d’oeuvres contains two types of filled puff pastry –chicken and shrimp. The entire platter has 15 pastries –8 chicken and 7 shrimp. From the outside, the pastries appear identical and they are randomly distributed on the tray. Choose three at random. What is the probability that a) all are chicken; b) all are shrimp; c) all have the same filling?

*Solution:*

*a* = 8 = the number of chicken pastries

*b* = 7 = the number of shrimp pastries

*a* + *b* = 15 = the total number of pastries

*X* = 3 = the number of pastries selected

In this case, since all are the same, *n* is also 3, so *n – X* = 0.

1. *a*C*X* = 8C3 = 56, *b*C*n*-*X* = 7C0 = 1, *a*+*b*C*n* = 15C3 = 455
2. aCX = 7C3 = 35, bCn-X = 8C0 = 1, a+bCn = 15C3 =455
3. + 0.123077 + 0.076923 = 0.2000 when rounded to four decimal places.

### Geometric Distribution

The **geometric distribution** is used when an experiment that has two outcomes is repeated until a successful outcome is obtained.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a probability experiment if it satisfies the following requirements:

1. Each \_\_\_\_\_\_\_ has two outcomes that can be classified as either success or failure.
2. The outcomes are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of each other.
3. The probability of a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is the same for each trial.
4. The experiment continues until a success is obtained.

### Formula for Calculating Probabilities Using the Geometric Distribution

The probability of getting the first success on the nth trial is P(n)=p(1-p)^(n-1), where n = 0, 1, 2, ..., where p is the probability of a success in each trial of a binominal experiment and n is the number of the trial in which the first success occurs.

### Example 5-23. Probability of Hitting Target

Arianna shoots arrows at a target and hits the bulls-eye about 40% of the time. Find the probability that she will hit the target on the third shot.

*Solution:*

*p* = 0.4

*n =* 3 (the first bullseye is on the third shot)

*P*(3) = (0.4)(1 - 0.4)3-1 = 0.4(0.6)2 = 0.144